— Exercises Week 7 —

Exercise 1) Show that a group of translations $\Gamma < \mathrm{Aut}(\mathbb{C})$ that acts prop. disc. on $\mathbb{C}$ must be isomorphic to $\mathbb{Z}$ or $\mathbb{Z}^2$

(Hint: consider the elements $g \in \Gamma$ that displace the origin the least.)

Exercise 2) Check by hand that the cylinder of circumference $\lambda \in \mathbb{R}$ is conformally equivalent to $\mathbb{C}^*$

Exercise 3) Show using $\mathrm{Aut}(\mathbb{S}^2) \cong \mathbb{P}S\ell(2,\mathbb{C})$ that a loxodromic $F \in \mathrm{Aut}(\mathbb{D})$ restricts to a homeo of $\mathbb{D}$ iff it is hyperbolic with $\mathrm{Fix}(F) \subset \partial\mathbb{D}$.

(Hint: it might be convenient to conjugate $\mathbb{D}$ onto the upper half-plane, so that $\mathrm{Aut}(\mathbb{D})$ becomes $\mathbb{P}S\ell(2,\mathbb{R})$)

Recall that a subset Y of a topological space X is discrete if it has no accumulation points (i.e. $\nexists\, x \in X$ and $y_n \in Y \setminus \{x\}$ such that $y_n \to x$)

Exercise 4) Show that $\Gamma < \mathrm{Aut}(\mathbb{D}) < \mathrm{Aut}(\mathbb{S}^2) \cong \mathbb{P}S\ell(2,\mathbb{C})$ acts prop. disc. on $\mathbb{D}$ iff Γ is discrete in $\mathbb{P}S\ell(2,\mathbb{C})$

(2×2 matrices is equipped with the topology making it homeo to $\mathbb{C}^4$. $S\ell(2,\mathbb{C})$ has the s.set topology and $\mathbb{P}S\ell(2,\mathbb{C})$ the quotient top.

Alternatively matrices in $S\ell(2,\mathbb{C})$ converge $A_n \to A$ iff their coefficients do)

Note that this is not the case for $\Gamma < \mathrm{Aut}(\mathbb{C})$ or $\Gamma < \mathrm{Aut}(\mathbb{S}^2)$ acting on $\mathbb{C}$ and $\mathbb{S}^2$ respectively.

here we mean "$\forall x\ \exists U$ nbhd of x s.t. $\{\gamma \in \Gamma \mid \gamma(U) \cap U \neq \emptyset\}$ is finite")